

Answer The Following Questions. CALCULATORS ARE NOT ALLOWED.

1. a) Find  $\lim_{x \rightarrow +\infty} (x - \sqrt{x^2 - 3x})$ , if it exists (3 marks)

b) Use the Sandwich Theorem to evaluate

$$\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{3\sqrt{x}}\right) \quad (3 \text{ marks})$$

2. Let

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x \leq 0 \\ Ax + B & \text{if } 0 < x \leq 3 \\ \frac{x^2 - 9}{x - 3} & \text{if } 3 < x \end{cases}$$

Find the constants A and B such that  $f$  is continuous for all real numbers. (6 Marks)

3. a) Find the equation of the tangent line to the graph of  $x^3 + y^3 - 4xy = 0$  at the point  $(2, 2)$ . (3 Marks)

b) Find the average value of the function  $f(x) = \frac{1}{(2x+1)^{2/3}}$  over the interval  $[0, 13]$ . (3 Marks)

4. Evaluate the following integrals

a)  $\int \frac{1}{\sqrt{x} (\sqrt{x} + 1)^3} dx$

b)  $\int_{\pi/4}^{\pi/3} \frac{1 + \sin x}{\cos^2 x} dx$   
(6 Marks)